Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_

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**End Semester Examination – Nov/Dec – 2018**

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| **Code :** | **14AE2033** | **Duration :** | **3hrs** |
| **Sub. Name :** | **ADVANCED SPACE DYNAMICS** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | If two point masses m1 and m2 are acted upon only by the mutual force of gravity between them, find the motion of the centre of mass. Prove that the centre of mass moves with a constant velocity in a straight line. | CO1 | 10 |
| b. | Define a central orbit. Prove that angular momentum is a constant vector. Prove that the motion takes place in a plane. | CO1 | 10 |
| (OR) | | | | |
| 2. |  | Define Lambert’s problem. Derive Lambert theorem analytically. | CO1 | 20 |
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| 3. |  | Derive equations of motion for planar restricted three body problem in synodic (rotating) coordinate system. Write the two equations to find the locations of the five equilibrium points. Derive Jacobi integral. Derive the fifth-degree algebraic equation to find the location ofthe collinear Lagrangian point L1. | CO2 | 20 |
| (OR) | | | | |
| 4. |  | To study the motion near the equilibrium points, expand the force function Ω around a Lagrangian point. Find the linearized variational equation of motion in two dimensions.  Derive the fourth-degree characteristic equation  λ4 + (4 - Ωxx- Ωyy)λ2 + {ΩxxΩyy- (Ωxy)2}=0,  at the Lagrangian points. | CO2 | 20 |
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| 5. | a. | Construct the principal system of co-ordinates at the equilateral point L4 to show the angle α, which the major-axis of the elliptic orbit makes with the ξ-axis for mass ratio μ = 0.15 in the restricted three-body problem. Write the expression as function of μ to obtain the angle α. | CO2 | 6 |
| b. | Prove that the second-order derivatives at the equilateral point L4 are  Ωxx= 3/4, Ωxy = 3.31/2 (μ - 1/2)/2, Ωyy = 9/4. | CO2 | 8 |
| c. | Using these values of partial derivatives, prove that the characteristic equation is  λ4 + λ2 + 27μ (1 - μ)/4 = 0. | CO2 | 6 |
| (OR) | | | | |
| 6. | a. | Find the second-order derivatives at the collinear points. | CO2 | 9 |
| b. | Find the value of the critical mass μ0 = 0.0385..at the equilateral points.  Prove that all the 4 roots are pure imaginary at the equilateral pointsμ0. | CO2 | 11 |
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| 7. | a. | Explain periodic orbits. | CO1 | 4 |
| b. | Explain n-body problem. Derive the 10 integrals of n-body problem. Write the equations of motion of general three-body problem. | CO1 | 16 |
| (OR) | | | | |
| 8. | a. | Derive Hamilton’s equations of motion for two-body problem in spherical polar coordinates. | CO1 | 10 |
| b. | Define extended phase space. Consider a dynamical system with two  degree of freedom with its Hamiltonian independent of time. Consider  a canonical transformation given by  W3 = - [p1f1(Q1, Q2) + p2 f2(Q1, Q2).  Use it to find the old coordinates and new momenta. In the extended 6-dimensional phase space, write the equations of motion. | CO1 | 10 |
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|  | | **Compulsory**: |  |  |
| 9. | a. | Prove that the following transformations are canonical.  (i) Q = (q2 + p2)/2, P = - tan-1 (q/p);  (ii) Q = q tan p, P = log (sin p);  (iii) Q= log{(sin p)/q}; P=q cot p. | CO1 | 12 |
| b. | Prove that the Hamiltonian of a harmonic oscillator  H = (p12 + p22)/2 + ω2(x12 + x22)/2,  with the help of the generating function  reduces to the form . | CO1 | 8 |